ON THE RELATION BETWEEN RAINFALL AND STREAM FLOW—III

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PART I

This article of the series deals with the evaporation which takes place after a rain stops, and its effect on stream flow.

Throughout, as in the first two papers, the rate of rainfall, the condition of the soil, and the velocity of the water are considered to be constant.

In the first section of Part II, equations are given which show the effect of evaporation on run-off. (As previously explained, for sufficiently small drainage areas the volume of rate of run-off can be regarded as synonymous with the discharge.) In the second section, equations are developed which give the discharge from a rectangle when the effect of evaporation is considered; in this section the rate of evaporation (or evaporativity) is treated as constant. In the third section, equations are developed which give the discharge from a drainage area of any shape, such as discussed in section 4 of the second paper, and where the rate of evaporation (or evaporativity) is any function of time. In the fourth section the diurnal variation of rate of run-off is discussed. In the fifth and last section a few hypothetical hydrographs are computed from which one can judge of the magnitude of the effect of evaporation on the discharge.

A few definitions will here be given and explained. First of all it is essential when speaking of "rainfall" or "run-off' or "evaporation" to keep clearly in mind exactly what is meant. Each of these three terms could be interpreted as representing a *depth* of water. Again each could be interpreted as representing a *volume* of water, by multiplying the *depth* by the area of a parcel of ground. Each of these terms could also be interpreted as representing a *change in depth per unit time;* in this sense, each term represents the first derivative of a *depth* with respect to time. Again each term could be interpreted as representing a *change in volume per unit time;* in this sense the term represents the first derivative of a *volume* with respect to time. Thus all told, four separate and distinct interpretations can be placed on each of the terms "rainfall", "run-off", and "evaporation". It is imperative to distinguish clearly between these meanings; and for this reason the practice of prefixing the words "rate of", "volume of", or "volume of rate of" to the basic word, as was done in the first paper in connection with "rainfall" and "run-off", will be followed here for "evaporation."

The volume of water (not of water vapor) which is evaporated from a given area in a given interval of time is here denoted the *volume of evaporation*. Like volume of rainfall, volume of evaporation will be measured in mile-inches. It should be clearly understood that the volume of evaporation as here defined includes both the water evaporated from the ground, whether directly or through plants, and also the water evaporated from any free water surfaces which the drainage area in question

may contain as lakes, ponds, and streams draining the area. Naturally for some studies it is necessary to distinguish between evaporation directly from the ground, evaporation through plants, etc., but it is not believed that these distinctions are necessary to forecast flood crests accurately.

The volume of water which is evaporated per unit time from a given area at any given time is designated the *volume of rate of evaporation*.

The volume of rate of evaporation is volume per unit time and therefore will be measured in mile-inches per hour. Mathematically, the volume of rate of evaporation is the first derivative of volume of evaporation with respect to time.

The volume of water which is evaporated from a given region in a given interval of time per unit area is here called the *evaporation*. Evaporation is volume divided by area, i. e., it is a length, and will be measured in inches. The volume of water which is being evaporated per unit time, from a given region at any given time, per unit area, is the *rate of evaporation*. The rate of evaporation is volume per unit time divided by area, i. e., length per unit time, and will therefore be measured in inches per hour. Mathematically the rate of evaporation is the first derivative of evaporation with respect to time.

It is well known that, other things being equal, the rate of evaporation is greater when there is a strong wind than when there is no wind; likewise, the rate of evaporation is greater when the air is warm than when the air is cold; greater when the sun is shining than when the sky is overcast; and greater when the relative humidity is low than when it is high. Recognizing these facts, consider the following two situations: First, a piece of ground during one year under specified conditions of culture, temperature, wind, etc., shortly after a heavy rain of long duration; and second, the same piece of ground during another year at the *same* specified conditions of culture, temperature, wind and all other pertinent things with this one difference: A long dry spell has been in progress. Clearly, the rate of evaporation will be greater in the first case than in the second. We can therefore say that the rate of evaporation *may* be dependent upon the amount of water remaining with the soil. That the rate of evaporation is not always dependent upon the amount of water remaining with the soil can be made clear by considering a slough. As long as there is water in the slough the rate of evaporation will be solely dependent on the wind, temperature and other weather elements and *not* upon the amount of water in the slough.

Undoubtedly experiments are necessary to determine just when the rate of evaporation becomes dependent upon the amount of water remaining with the soil; but it would seem that when soil is water-logged, as it may be occasionally, the rate of evaporation would not be so dependent.

To take account of the fact that the rate of evaporation may be dependent upon the amount of water remaining with the soil, it is necessary to introduce two additional terms: The volume of water which would be evaporated from a given region, were an unlimited supply of water available in a given interval of time per unit area is here called the *evaporativity*. The volume of water which would be evaporated per unit time from a given region were an unlimited supply of water available at any given time per unit area is designated the *rate of evaporativity*. Mathematically, the rate of evaporativity is the first derivative of evaporativity with respect to time. Evaporativity is measured in the same units as evaporation; and rate of evaporativity is measured in the same units as rate of evaporation.

The use here of the term evaporativity, as distinguished from the term evaporation, agrees with the recommendations of a committee of the Section of Hydrology of the American Geophysical Union.[1]

The distinction between evaporation and evaporativity can be explained in a different way: By multiplying the depth of rainfall by the area of a drainage area the volume of rainfall can be computed; if the depth of rainfall varies from place to place within the drainage area, the volume of rainfall should be computed by an integration process. In a similar way the volume of discharge on the stream which drains the region can be computed. If the time of beginning and the time of ending of the time interval be so chosen that the amount of water remaining with the soil is the same at the end as at the beginning, and the volume of discharge is equal to the volume of run-off, then substracting the volume of discharge from the volume of rainfall we obtain a remainder which represents the volume of evaporation and is the actual amount of water which was lost from the drainage area during the given interval by evaporation processes. Now suppose a number of evaporation pans be located in this drainage area, and the depth of water lost from each pan to the air by evaporation in this interval of time be measured and recorded. Then by an appropriate integration process (or simple multiplication if no variation is observed from pan to pan within the region) the total volume of water that would be lost to the air over the region in the interval *were an unlimited supply of water kept available* could be computed. It is evident that for most regions of the earth's land surface the volume of evaporation computed from evaporation pan observations would exceed the volume of water obtained by subtracting the volume of run-off from the volume of rainfall. In keeping with the definitions just given, we say that in general observations obtained from evaporation pans represent evaporativity and very seldom represent evaporation.

All the terms defined in the first three articles, together with the units in which they are expressed, are tabulated below.

Throughout the first three sections of the present article, evaporation has been approached from two different points of view, viz. (1) that evaporation is independent of the amount of water remaining with the soil; and (2) that it is so dependent. Thus there are two sets of equations: One set applying to the first condition; the other to the second. Both sets have been derived for the sake of completeness. It would seem that in the eastern part of the United States the soil seldom becomes water-logged except in winter and early spring, and at these times of the year the rate of evaporation is very low anyway and probably could safely be neglected. The soil is not likely to become water-logged at other times of the year even

¹ These terms appear on p. 403 of *Transactions of the American Geophysical Union* (1935).

though the most intense rains do occur in summer. Hence, when it is necessary to consider evaporation, it would seem that only the equations derived on the second assumption above would be required. Further comments on this question will be made when the theory here developed is applied to actual observations.

TERMS RELATED TO RUN-OFF

Term	Symbols	Units in which expressed
Run-off	$\int_0^t z\,dt$	Inches.
Rate of run-off	z	Inches per hour.
Volume of run-off	$\int_0^t A z\,dt$ or $\int\int\int z\,dx\,dw\,dt$	Mile-inches.
Volume of rate of run-off	$Az = Z$ or $\int\int z\,dx\,dw$	Miles-inches per hour.

TERMS RELATED TO RAINFALL

Term	Symbols	Units in which expressed
Rainfall	R	Inches.
Rate of rainfall	$\dfrac{dR}{dt} = r$	Inches per hour.
Volume of rainfall	AR or $\int\int\int r\,dx\,dw\,dt$	Mile-inches.
Volume of rate of rainfall	Ar or $\int\int r\,dx\,dw$	Mile-inches per hour.

TERMS RELATED TO EVAPORATION

Term	Symbols	Units in which expressed
Evaporation	$\int_0^t E\,dt$	Inches.
Rate of evaporation	E	Inches per hour.
Volume of evaporation	$\int_0^t A E\,dt$ or $\int\int\int E\,dx\,dw\,t$	Mile-inches.
Volume of rate of evaporation.	AE or $\int\int E\,dx\,dw$	Mile-inches per hour.
Evaporativity	$\int_0^t \mathcal{E}\,dt$	Inches.
Rate of evaporativity	$\mathcal{E}$	Inches per hour.

TERMS RELATED TO DISCHARGE

Term	Symbols	Units in which expressed
Volume of discharge	$\int_0^t y\,dt$	Mile-inches.
Discharge	y	Mile-inches per hour.
Rate of discharge	dy/dt	Mi. in. hr⁻¹
Discharge tendency	d^2y/dt^2	Mi. in. hr⁻²

The various assumptions which are made from time to time throughout this series of articles are of two very different kinds: First are the *special* assumptions, of which may be mentioned the assumptions that the drainage area is rectangular, the rate of rainfall is constant, the velocity of the water is constant, etc. These special assumptions will one by one be removed; they are made in order to have the early theoretical treatment tractable. Second are the *fundamental* assumptions. The first fundamental assumption, made in the first paper, is that the rate of run-off from a parcel of ground at any given time is directly proportional to the amount of water remaining with the soil at that time. No fundamental assumptions were made in the second paper. In this third paper a second and a third assumption are made: The second is: When the soil is not water-logged, the evaporation at any given instant is directly proportional to the product of the amount of water remaining with the soil by the evaporativity at that instant. The third is: When transpiration is constant, the rate of run-off at any given time is directly proportional to the amount of water remaining with the soil at that time; when the latter is constant, the rate of run-off at any given time is inversely proportional to the transpiration.

These fundamental assumptions are believed to be plausible, and to be close approximations to, if not precisely, what occurs in nature. They have been made not in order to have the mathematical treatment tractable but because no exact observations of these phenomena are available.

It will be pointed out in the fourth section of this paper that although the third fundamental assumption must be considered when dealing with the rate of run-off, it can safely be disregarded when dealing with stream flow from a drainage area of appreciable size.

In regard to the question of evaporation and its effect on stream flow, despite the fact that for the eastern part of the United States only about one-fourth of the mean annual rainfall runs off while approximately three-fourths evaporates; nevertheless, in a rain of sufficient heaviness and intensity to cause a flood the part which evaporates before the time of a flood crest is relatively small and usually has a negligible effect on the height of the flood crest. The hydrographs in the fifth section clearly show this.

PART II

SECTION 1: RUN-OFF CORRECTED FOR EVAPORATION

The symbols in this paper have the same meanings given them in the preceding papers with this exception: $\phi(t)$ henceforth represents *depth* of water remaining with the soil (not the *volume* as stated on page 318 of the first paper).

Equation (2) of paper I will now be generalized.

A first generalization will be developed on the assumption that the amount of water remaining with the soil is sufficiently great for the rate of evaporation to be independent thereof. The rate of evaporation is a function of the wind, temperature, relative humidity, and possibly other things also, but for the present purpose the rate of evaporation can be regarded simply as a function of time. As evaporation may be considered to begin at the instant the rain stops, that is, when $t=t_0$ (or when $t'=t-t_0=0$) let this function be $E(t')$. After the rain stops, the volume of water remaining with the soil at any given time is equal to the volume which fell as rain less the volume which ran off while it was raining, less the volume which has run off during the interval since the rain stopped, less the volume which has evaporated during this same interval. Expressed in symbols:

$$A\phi(t) = \int_0^{t_0} Ardt - \int_0^{t_0} Azdt - \int_{t_0}^{t} Azdt - \int_{t_0}^{t} AE(t')dt.$$

In accordance with the first fundamental assumption, $\phi(t)$ can be replaced by cz, and on doing this and differentiating we get

$$cAdz = -Azdt - AE(t')dt.$$

Clearly $dt = dt'$; and dividing by $cAdt'$ and transposing,

$$\frac{dz}{dt'} + \frac{1}{c}z = -\frac{1}{c}E(t'),$$

a linear differential equation of the first order.[1] The integrating factor is

$$e^{\int \frac{dt'}{c}} = e^{\frac{t'}{c}},$$

hence

$$e^{\frac{t'}{c}}\frac{dz}{dt'} + \frac{1}{c}e^{\frac{t'}{c}}z = -\frac{1}{c}E(t')e^{\frac{t'}{c}},$$

and

$$\left[ze^{\frac{t'}{c}}\right]_0^{t'} = \int_0^{t'} -\frac{1}{c}E(t')e^{\frac{t'}{c}}dt.$$

Now $z = z_0$ when $t' = 0$, that is, at the time the rain stops; hence, evaluating the left member, transposing and multiplying by $e^{-\frac{t'}{c}}$, we have finally:

$$z = e^{\frac{-t'}{c}}\left[z_0 - \frac{1}{c}\int_0^{t'} E(t')e^{\frac{t'}{c}}dt'\right]. \tag{C-2}$$

Equation (C-2) is the first generalized form of equation (2). It should be noted that when $E(t') \equiv 0$ then equation (C-2) reduces to equation (2) as it should.

A second generalized form of equation (2) will next be derived. Suppose that at the time the rain stops the amount of water remaining with the soil is not sufficiently great for the rate of evaporation to be independent thereof. In this case the rate of evaporativity can be regarded as a function of time. Let this function be $\mathcal{E}(t')$. Let $\phi(t')$ be the depth of water remaining with the soil at time t'; then in accordance with the second fundamental assumption, the evaporation at time t' is given by $\kappa\{E(t')\} = \phi(t')\,\mathcal{E}(t')$, where κ is a constant of proportionality. On substituting this value of $E(t')$ in the linear differential equation given above, we get

$$\frac{dz}{dt'} + \frac{1}{c}z = -\frac{1}{c\kappa}\phi(t')\,\mathcal{E}(t').$$

As before, in accordance with the first fundamental assumption $\phi(t')$ can be replaced by cz; then we have

$$\frac{dz}{dt'} + \frac{1}{c}z = -\frac{1}{\kappa}z\,\mathcal{E}(t').$$

Clearly, this can be written in the form

$$\frac{dz}{z} = -\left(\frac{1}{c} + \frac{1}{\kappa}\mathcal{E}(t')\right)dt';$$

and by integrating

$$\log z = -\frac{t'}{c} - \frac{1}{\kappa}\int\mathcal{E}(t')dt' + \text{const.}$$

Then from the definition of a logarithm and the fact that $z = z_0$ when $t' = 0$, we have finally

$$z = z_0 e^{-\frac{t'}{c} - \frac{1}{\kappa}\int_0^t \mathcal{E}(t')dt'} \tag{C-2f}$$

Equation (C-2f) is the second generalized form of equation (2). When $\mathcal{E}(t') \equiv 0$ then equation (C-2f) reduces to equation (2) as it should. It is important to bear in mind that equation (C-2f) was derived on the assumption that the amount of water remaining with the soil at the time the rain stops is not great enough for the rate of evaporation to be independent thereof.

As already explained, c is a time constant and is measured in hours. The constant κ is a depth (length) constant and is measured in inches. It represents the depth of water which is just sufficient to saturate the soil enough to make the rate of evaporation independent of the amount of water remaining with the soil. This depth,[2] necessary to produce just this degree of saturation, varies with the condition of the soil at the beginning of the rain. If a rain is so gentle that all the water soaks into the soil (that is, there is no *surface* run-off), and also lasts just long enough to bring about this degree of soil saturation, then κ is equal to cz_0.

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[1] It may be noted that both equations (1) and (2) in the first paper could have been derived by making use of some of the elementary theory of differential equations, instead of the more elementary processes actually used.

[2] It should be emphasized that stream flow can be considered as arising from three causes, namely: (1) Surface run-off from the last rain in the river basin; (2) ground water run-off from this last rain; (3) ground water run-off from antecedent rains; and in all the statements here being made about the constants c and κ the stream flow due to the third cause is ignored. In other words, κ represents the depth as stated only insofar as the stream flow from a *single* rain is being represented.

From the above it is evident that κ can be regarded as known; and it is known at the beginning of the rain. If the rain is so heavy and so long that after the rain stops equation (C–2) applies, then this equation should be used only for a limited range. The variable z in equation (C–2) should be replaced by the constant $\frac{\kappa}{c}$ and the resulting equation solved for t; let this value of t be $\bar{t}$. Then $\bar{t}$ represents the time at which the amount of water with the soil is such that thereafter the rate of evaporation is dependent on the amount which remains. It is evident then that equation (C–2) applies on the range $0 \leq t' \leq \bar{t}$ (or $t_0 \leq t \leq \bar{t} + t_0$) and equation (C–2f) applies on the range $\bar{t} \leq t' \leq \infty$ (or $\bar{t} + t_0 \leq t \leq \infty$). If the rain is such that when the rain stops, the evaporation is dependent on the amount of water remaining with the soil, then equation (C–2f) applies on the range $0 \leq t' \leq \infty$.

In the first paper the discharge equations were verified by integrating them between the limits of the respective ranges over which they apply, and ascertaining that the sum of the integrals thus obtained is equal to the volume of rainfall which occurs over the drainage area. The same principle will now be used to verify the run-off equations just obtained by showing that the rainfall is equal to the sum of the run-off and the evaporation. First, suppose the amount of water remaining with the soil at the time the rain stops is small enough for the evaporation to be dependent on this amount. The depth of water remaining with the soil at the time the rain stops is

$$\phi(t_0) = \int_0^{t_0} r \, dt - \int_0^{t_0} r \left(1 - e^{-\frac{t}{c}}\right) dt$$

$$= rc\left(1 - e^{-\frac{t_0}{c}}\right).$$

The sum of the run-off and the evaporation is

$$\int_{t_0}^{\infty} z \, dt + \int_0^{\infty} E(t') \, dt' = \int_0^{\infty} z_0 e^{-\frac{t'}{c} - \frac{1}{\kappa}\int_0^{t'} \varepsilon(t')dt'} dt'$$

$$+ \int_{t_0}^{\infty} \frac{1}{\kappa} \phi(t') \, \varepsilon(t') dt'$$

$$= \int_0^{\infty} z_0\left(1 + \frac{c}{\kappa}\varepsilon(t')\right) e^{-\frac{t'}{c} - \frac{1}{\kappa}\int_0^{t'} \varepsilon(t')dt} dt' =$$

$$cz_0\left[-e^{-\frac{t'}{c} - \frac{1}{\kappa}\int_0^{t'} \varepsilon(t')dt'}\right]_0^{\infty} = cz_0 = rc\left(1 - e^{-\frac{t_0}{c}}\right),$$

which is equal to the depth remaining with the soil at the time the rain stops.

Suppose now that $rc\left(1 - e^{-\frac{t_0}{c}}\right)$ is so great that for a while the rate of evaporation is independent of this depth. In this case equation (C–2) is integrated between the limits 0 and $\bar{t}$. The evaporation plus the runoff from the time the rain stops to the time $\bar{t}$ is

$$\int_0^{\bar{t}} E(t') \, dt' + \int_0^{\bar{t}} e^{-\frac{t'}{c}}\left[z_0 - \frac{1}{c}\int_0^{\bar{t}} E(t')e^{\frac{t'}{c}} dt'\right] dt' =$$

$$\int_0^{\bar{t}} E(t') \, dt' + z_0 \int_0^{\bar{t}} e^{-\frac{t'}{c}} dt' + \int_0^{\bar{t}}\left(-\frac{1}{c}e^{-\frac{t'}{c}}\right)\left(\int_0^{t'} E(t')e^{\frac{t'}{c}} dt'\right) dt' =$$

$$\int_0^{\bar{t}} E(t') \, dt' + cz_0\left(1 - e^{-\frac{\bar{t}}{c}}\right) + \left[e^{-\frac{t'}{c}}\int_0^{t'} E(t')e^{\frac{t'}{c}} dt' - \int_0^{t'} E(t')dt'\right]_0^{\bar{t}}$$

$$= cz_0\left(1 - e^{-\frac{\bar{t}}{c}}\right) + e^{-\frac{\bar{t}}{c}}\int_0^{\bar{t}} E(t')e^{\frac{t'}{c}} dt'.$$

The depth remaining at time t is therefore

$$cz_0 e^{-\frac{t}{c}} - e^{-\frac{t}{c}}\int_0^{t'} E(t')e^{\frac{t'}{c}} dt'$$

and from the definition of $\bar{t}$ this must equal κ. The evaporation plus the runoff from the time $\bar{t}$ till all water has disappeared from the soil is

$$\int_{\bar{t}}^{\infty} z \, dt' + \int_{\bar{t}}^{\infty} E(t') dt' = \kappa \int_0^{\infty}\left(1 + \frac{c}{\kappa}\varepsilon(t')\right) e^{-\frac{t'}{c} - \frac{1}{\kappa}\int_0^{t'} \varepsilon(t')dt'} dt'$$

$$= \kappa \text{ as it should.}$$

SECTION 2: DISCHARGE FROM RECTANGLE CORRECTED FOR EVAPORATION

Equations will now be obtained for a rectangular drainage area, under the assumption that the amount of water remaining with the soil at the time the rain stops is not great enough to make the evaporation independent thereof. It is further assumed in this simple case that the rate of evaporativity is constant.

Undoubtedly there is a small amount of evaporation while the rain is in progress, but it is here assumed to be negligible, and for this reason no modification is necessary in equations (3) and (4). Equation (5) applies on the range $t_0 + \frac{L}{v} \leq t \leq \infty$, and it follows by reasoning similar to that given in the first paper that on this range the discharge is obtained by integrating the volume of rate of runoff over the entire drainage area. Making use of equation (C–2f) the volume of rate of runoff from the infinitesimal strip, $W dx$, at distance x above the gaging station is given by

$$W z \, dx = W z_0 e^{-\left(t' - \frac{x}{v}\right)/c - \frac{1}{\kappa}\varepsilon\left(t' - \frac{x}{v}\right)} dx$$

$$= W z_0 e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)\left(t' - \frac{x}{v}\right)} dx$$

On integrating $W z \, dx$ between the limits 0 and L we get

$$y = W z_0 \frac{c\kappa v}{\kappa + c\varepsilon}\left[e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)\left(t' - \frac{L}{v}\right)} - e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)t'}\right]$$

Since $z_0 = r\left(1 - e^{\frac{-t_0}{c}}\right)$ and $t' = t - t_0$ we have finally

$$y = W r c v\left(1 - e^{\frac{-t_0}{c}}\right)\frac{\kappa}{\kappa + c\varepsilon}\left[e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)\left(t - t_0 - \frac{L}{v}\right)} - e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)(t - t_0)}\right]$$

This is equation (26). When $\varepsilon = 0$ equation (26) reduces to equation (5) as it should.

If $t_0 > \frac{L}{v}$ then for the range $t_0 \leq t \leq t_0 + \frac{L}{v}$ equation (6) requires modification. It follows by reasoning similar to that of the first paper, and that used in obtaining equation (26), that on this range the discharge is

$$y = \int_{x_0}^{L}\left[W r \, 1 - e^{-\frac{1}{c}\left(t - \frac{x}{v}\right)}\right] dx + \int_{\cdot}^{x_0} W z_0 e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)\left(t' - \frac{x}{v}\right)} dx.$$

On performing the integration indicated, recalling that $x_0 = (t - t_0)v$, and simplifying, we get finally

$$y = \left[W r \, L - (t - t_0)v + cv\left(e^{-\frac{t_0}{c}} - e^{-\frac{1}{c}\left(t - \frac{L}{v}\right)}\right)\right]$$

$$+ W r c v\left[\left(1 - e^{-\frac{t_0}{c}}\right)\frac{\kappa}{\kappa + c\varepsilon}\left\{1 - e^{-\left(\frac{1}{c} + \frac{\varepsilon}{\kappa}\right)(t - t_0)}\right\}\right] \quad (27)$$

When $\mathcal{E}=0$ equation (27) reduces to equation (6) as it should.

If $t_0 < \dfrac{L}{v}$, equation (9) requires modification for the range $t_0 \leqq t \leqq \dfrac{L}{v}$. By reasoning similar to that just used, as well as from the explanations in the first paper, it follows that on this range the discharge is given by

$$y = Wrv\left[t_0 + c\left(1 - e^{-\frac{t_0}{c}}\right)\left(\frac{\kappa}{\kappa + c\mathcal{E}}\left\{1 - e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)(t-t_0)}\right\} - 1\right)\right], \quad (28)$$

and this equation reduces to equation (9) when $\mathcal{E}=0$.

As previously explained, the derivatives, with respect to t, of equations (3) and (4) are everywhere positive. It follows readily that the derivative of equation (28) is everywhere positive, and that for equation (26) is everywhere negative. Hence whether $t_0 \gtreqless L/v$ the time of the flood crest is obtained by equating the derivative of equation (27) with respect to time to zero and solving. If t_c represent the time of the crest, the equation obtained by equating the derivative of equation (27) to zero cannot be solved explicitly for t_c. However, in the case of y_c the maximum discharge, we can resort to the device, which was used several times in the second paper, of multiplying equation (27) by $e^{\frac{t}{c}}$, differentiating the equation thus obtained, setting $dy/dt = 0$ and finally multiplying by $ce^{-\frac{t}{c}}$.

Or doing this we have

$$y_c = Wr\left[L - (t_c - t_0)v + cv\left(1 - e^{-\frac{t_0}{c}}\right)\right.$$
$$\left.\left\{\frac{\kappa}{\kappa + c\mathcal{E}}\left(1 + \frac{c\mathcal{E}}{\kappa}e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)(t-t_0)}\right) - 1\right\}\right] \quad (29)$$

When $\mathcal{E}=0$ equation (29) reduces to equation (8) as it should.

We are now ready to consider the case when the amount of water remaining with the soil at the time the rain stops is so great that the rate of evaporation is independent thereof. Here we further assume that the rate of evaporation, from the time the rain stops to the time the amount of water remaining with the soil is small enough to make the rate of evaporation dependent thereon, is constant.

If E is constant, as we are now assuming, equation (C—2) takes the form $z = e^{-\frac{t'}{c}}\left[z_0 - E\left(e^{\frac{t'}{c}} - 1\right)\right]$, and the contribution to the discharge from the infinitesimal strip Wdx at distance x above the gaging station is $W\left[z_0 e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)}\right.$ $\left. + E\left(e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)} - 1\right)\right]dx$. If $t_0 < \dfrac{L}{v}$, then on the range $t_0 \leqq t \leqq \dfrac{L}{v}$ equation (9) requires modification; and on this range the discharge is given by

$$y = \int_{x_0}^{tv} Wr\left[1 - e^{-\frac{1}{c}\left(t - \frac{x}{v}\right)}\right]dx$$
$$+ \int_0^{x_0} W\left[z_0 e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)} + E\left(e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)} - 1\right)\right]dx$$
$$= Wrv\left[t_0 + ce^{-\frac{t}{c}}\left(1 - e^{-\frac{t_0}{c}}\right)\right]$$
$$- WEv\left[t - t_0 + c\left(e^{-\frac{1}{c}(t'-t_0)} - 1\right)\right]. \quad (30)$$

Equation (30) holds on the range $t_0 \leqq t \leqq L/v$ with the further restriction that $z > \dfrac{\kappa}{c}$. When $\mathcal{E}=0$, equation (30) reduces to equation (9).

If $\dfrac{L}{v} \leqq t \leqq t_0 + \dfrac{L}{v}$ and $t > t_0$, equation (6) must be changed. For this range (with the additional restriction that $z > \kappa/c$ of course) the discharge is given by

$$y = Wr\left[L - (t - t_0)v + cv\left\{1 + e^{-\frac{t}{c}}\left(1 - e^{\frac{L}{cv}} - e^{\frac{t_0}{c}}\right)\right\}\right]$$
$$- WEv\left[t - t_0 + c\left(e^{-\frac{1}{c}(t-t_0)} - 1\right)\right], \quad (31)$$

which reduces to equation (6) when E=0.

If $t_0 + \dfrac{L}{v} \leqq t \leqq \infty$, with the additional restriction that $z > \dfrac{\kappa}{c}$ of course, the discharge is given by

$$y = Wrcve^{-\frac{t}{c}}\left[e^{\frac{1}{c}\left(\frac{L}{v} + t_0\right)} - e^{\frac{L}{cv}} - e^{\frac{t_0}{c}} + 1\right]$$
$$- WE\left[L + cve^{-\frac{1}{c}(t-t_0)}\left(1 - e^{\frac{L}{cv}}\right)\right], \quad (32)$$

which reduces to equation (5) when E=0.

Suppose that $z = \dfrac{\kappa}{c}$ at time $t = t_0 + \bar{t}$, and suppose further $t_0 + \bar{t} \geqq t_0 + \dfrac{L}{v}$; then on the range $t_0 + \bar{t} \geqq t \geqq \infty$ the discharge is given by

$$y = \int_{(t'-\bar{t})v}^{L} W\left[z_0 e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)} + E\left(e^{-\frac{1}{c}\left(t' - \frac{x}{v}\right)} - 1\right)\right]dx$$
$$+ \int_0^{(t'-\bar{t})v} W\frac{\kappa}{c}e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)\left(t' - \frac{x}{v}\right)}dx. $$

On performing the integration and simplifying we finally get

$$y = Wrcv\left(e^{-\frac{1}{c}(t-t_0-L/v)} - e^{-\frac{\bar{t}}{c}} - e^{-\frac{1}{c}(t-L/v)} + e^{-\frac{1}{c}(\bar{t}+t_0)}\right)$$
$$+ WEcv\left(e^{-\frac{1}{c}(t-t_0-L/v)} - e^{-\frac{\bar{t}}{c}}\right) - WE\left(L - \{t - t_0 - \bar{t}\}v\right)$$
$$+ Wkv\frac{\kappa}{\kappa + c\mathcal{E}}\left(e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)\bar{t}} - e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)(t-t_0)}\right). \quad (33)$$

Equation (33) holds on the range $\bar{t} \leqq t' \leqq \bar{t} + \dfrac{L}{v}$. When $t = \bar{t} + t_0$ equation (33) reduces to the form

$$y = WEcve^{-\bar{t}/c}\left(e^{L/cv} - 1\right) - WEL +$$
$$+ Wrcve^{-\frac{1}{c}(\bar{t}+t_0)}\left(e^{\frac{1}{c}\left(\frac{L}{v}+t_0\right)} - e^{L/cv} - e^{t_0/c} + 1\right). \quad (33a)$$

When t is set equal to $\bar{t} + t_0$ in equation (32), the latter takes the form of equation (33a) also, as would be expected. When $t = \bar{t} + t_0 + L/v$ equation (33) reduces to the form

$$y = Wkv\frac{\kappa}{\kappa + c\mathcal{E}}\left(e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)\bar{t}} - e^{-\left(\frac{1}{c} + \frac{\mathcal{E}}{\kappa}\right)(\bar{t}+L/v)}\right) \quad (33b)$$

Equation (26) involves the factor $z_0 = r\left(1 - e^{-\frac{t_0}{c}}\right)$. If this factor be replaced by κ/c then equation (26) reduces to the

form of equation (33b) when $t=t+t_0+L/v$. This fact requires some explanation. If, at the beginning of a rain, κ be the depth of water (in inches) required to produce just enough saturation of the soil for the rate of evaporation to be independent of the water which remains, and if this rain fell and saturated the soil instantaneously, then $\kappa=R$, the depth of the rain. However, in nature there will never be such instantaneous rains; and the depth of rain required to produce this saturation of the soil is $\int_0^{t_0} r(1-e^{-t/c})\,dt$ less than rt_0; that is to say, the depth of rainfall, rt_0, must be diminished by the run-off while the rain is in progress, and κ is then equal to rc $(1-e^{-t_0/c})$, or in other words $\kappa=z_0c$ *provided* t_0 does not exceed the time required to produce *exactly* this degree of saturation. Clearly, in general, t_0 will either be less than or greater than this time. If t_0 is less than this time, the rate of evaporation is dependent on the amount which remains and therefore one of the equations (26), (27) or (28) applies. Each of these three equations involves the factor $z_0=r$ $(1-e^{-t/c})$. If, however, t_0 exceeds the time necessary for κ inches of rainfall to accumulate and remain with the soil, then equations (26), (27) and (28) are not at once applicable but instead one of equations (30), (31) or (32) applies; equation (32) does not in this case apply on the range $t_0+\dfrac{L}{v}\leqq t\leqq\infty$, because eventually the water remaining with the soil will diminish to a value that will make the rate of evaporation dependent on this amount. Of course equation (32) would apply on this range if the soil were impervious and no rain soaked *into* the soil, because as long as there were free water above an impervious soil the supply of water available insofar as the effect on the rate of evaporation is concerned could be regarded as unlimited. When the water remains with the soil long enough after the rain has stopped for the rate of evaporation to become dependent thereon an equation of the form of (26) applies; but in this case we cannot use the factor $z_0=r(1-e^{-t_0/c})$, because t_0 *exceeds* the time required for z_0 to become just equal to κ/c. Now $\bar{t}$ represents the time, after the rain stops, at which the water remaining with the soil has receded to a value such that the rate of evaporation will thereafter be dependent on the amount of water which remains. Hence when t_0 exceeds the value for κ to be equal to rc $(1-e^{-t_0/c})$, and if $\bar{t}>L/v$, then equation (32) applies on the range $t_0+L/v\leqq t\leqq t_0+\bar{t}$, equation (33) on the range $t_0+\bar{t}\leqq t\leqq t_0+\bar{t}+L/v$, and equation (26), after the factor $z_0=r$ $(1-e^{-t_0/c})$ is replaced by κ/c, applies on the range $t_0+\bar{t}+L/v\leqq t\leqq\infty$.

If both E and ε are equal to zero, which is practically equivalent to saying that the air is so still and cloudy and damp that there can be no evaporation, we can replace $\bar{t}$ by any value of t' whatever; and equation (33) then reduces to equation (5) as it should.

If $\bar{t}<L/v$ equations (30) and (31) will not apply on the ranges stated, since in this case z has receded to a value equal to or less than κ/c. Equations for this situation will not be obtained. Their derivation, if desired, is perfectly straight-forward, following the reasoning used in getting equation (33).

It can be shown that the derivatives of equations (32) and (33) with respect to t are everywhere negative; that is, the flood crest cannot be given by these equations. If we differentiate equation (31) with respect to t we get

$$dy/dt=Wrv\left[-1-e^{-t/c}\left(1-e^{L/cv}-e^{t_0/c}\right)\right]-WEv\left[1-e^{-\frac{1}{c}(t-t_0)}\right]$$

If we set this derivative equal to zero, t is replaced by t_c, the time of the flood crest. Solving for t_c, we get

$$t_c=c\left[\log\left\{r\left(e^{L/cv}+e^{t_0/c}-1\right)-Ee^{\frac{t_0}{c}}\right\}-\log\left\{r-E\right\}\right].\quad(34)$$

The maximum discharge, y_c, is obtained by using the same device as in deriving equation (29); and from equation (31) we get in this way

$$y_c=Wr[L-(t_c-t_0)v]-WE(t_c-t_0)v.\quad(35)$$

When $E=0$ equation (34) reduces to equation (7), and equation (35) to equation (8), as they should.

The question arises whether equation (30) furnishes, in certain cases, the maximum discharge. To consider this take equation (C-2). From this equation it is evident $z=0$ when $z_0=\dfrac{1}{c}\int_0^{t'}E(t')e^{t'/c}dt'$. Equation (C-2) is only applicable till the time $\bar{t}$ and since after the time represented by the solution of this last equation for t' equation (C-2) gives negative values for the run-off it is clear that $\bar{t}$ cannot possibly exceed the value obtained by solving this equation. If $E(t')$ is constant this equation becomes

$$z_0=E(e^{t'/c}-1)\quad\text{or}\quad e^{t'/c}=1+\frac{z_0}{E}\quad\text{whence}\quad t'=c\,\log\left(1+\frac{z_0}{E}\right).$$

Thus since equation (C-2) is not applicable after this time neither is equation (30) which was based on equation (C-2). The derivative of equation (30) is

$$\frac{dy}{dt}=Wrv\left(e^{t_0/c}-1\right)e^{-t/c}-WEv\left(1-e^{-\frac{1}{c}(t-t_0)}\right)$$

equate this to zero and solve: $t_c=c\,\log\left[\dfrac{r}{E}\left(e^{t_0/c}-1\right)+e^{t_0/c}\right]$

But equation (30) cannot possibly hold after

$$t=t_0+c\,\log\,(1+z_0/E).$$

In Nature, it will never even hold up to this value. Since $z_0=r(1-e^{-t_0/c})$ and $t_0=c\,\log\,e^{t_0/c}$ we have

$$t=c\,\log\left[e^{t_0/c}\left\{1+r/E\left(1-e^{-t_0/c}\right)\right\}\right].\quad\text{That is, }t_c\text{ representing}$$

the point where the derivative of equation (30) takes the zero value equals this value up to which equation (30) will never apply in Nature. Therefore equation (30) cannot have a maximum on the range for which it holds.

SECTION 3: DISCHARGE FROM ANY DRAINAGE AREA CORRECTED FOR EVAPORATION

The equations of section 2, wherein the width of the drainage area, and the rate of evaporation (or evaporativity) were considered constant, serve as a simple illustration of, and an introduction to, the underlying principles involved in correcting the discharge for evaporation in general. More general equations will now be developed in which neither the width of the drainage area nor the rate of evaporation (or evaporativity) is treated as constant.

The same mathematical restrictions are placed on $W(x)$, the function which represents the width of the drainage area, as in the second paper. The following mathematical restrictions are placed on the functions E (t') and ε (t') which represent the rate of evaporation and rate of evaporativity respectively:

1. They are identically zero when $t'<0$; in other words before the rain stops there is no evaporation.

2. They shall be single valued, except at points of discontinuity where they shall be two-valued.

3. They shall be everywhere finite.

4. They may assume an unlimited number of zero values, but may not assume any negative value.

5. They shall not have more than a finite number of discontinuities on a finite range.

When equations (C–2) and (C–2f) were derived, no explicit assumptions were made about the functions $E'(t')$ and $\mathcal{E}(t')$. Obviously these equations are valid if the functions are continuous; but it should be noted that they also hold when $E(t')$ and $\mathcal{E}(t')$ are subject merely to the more general restrictions just stated.

Each of the restrictions listed is justified by the physical nature of the problem. The fourth restriction may require special comment since evaporation is sometimes loosely spoken of as negative rainfall. To regard the evaporation which may take place while rain is in progress as negative rainfall is certainly logical. However, it does not seem to be convenient to treat evaporation in this way after the rain stops. Neither does it seem to be altogether convenient to treat the occasional light shower which may occur after the main heavy flood-causing rain as negative evaporation. Further developments may make such treatment desirable but for the present the fourth restriction is imposed.

In the following developments the discontinuity which may take place in the constant, c, during and after a rain is ignored, just as it has been previously. It was explained in the first paper that the constant c may be thought of as consisting of two parts: one, c', due to the fact that water soaks *into* the soil and the other, c'', due to the fact that water sometimes remains *upon* the soil.* The constant $c''=0$ when and only when the soil is saturated. The constant $c'=0$ when and only when the soil is not saturated and also the rate of rainfall is less than the infiltration rate. If the rate of rainfall exceed the infiltration rate and the soil is not saturated then neither c' nor c'' is zero. Both c' and c'' are never simultaneously zero. After an intense rain water may remain upon the soil for some time after the rain stops even though the soil is not saturated because the rate of rainfall exceeded the infiltration rate. In this case the rate of evaporation is independent of the depth of water remaining with the soil until and only until there is no water remaining upon the soil. At this instant there is a discontinuity in the constant c. The behavior of the constant c will be fully discussed in the fifth paper of this series.

Consider first the case where the soil contains so much water at the time the rain stops that the rate of evaporation is independent of the amount which remains. The rate of run-off from each infinitesimal strip, $W(x)dx$, above the gaging station is given by equation (C–2). By integrating this expression between the limits 0 and L, making due allowance for the time it takes for water to flow from where it fell as rain to the gaging station, we have

$$y=\int_0^L W(x)e^{-\frac{1}{c}\left(t'-\frac{x}{v}\right)}\left[z_0-\frac{1}{c}\int_0^{(t'-x/v)}E(t')e^{\frac{t'}{c}}dt'\right]dx$$

$$=e^{-t'/c}\left[z_0\int_0^L W(x)e^{x/cv}dx\right.$$

$$\left.-\int_0^L \frac{1}{c}W(x)e^{x/cv}\left\{\int_0^{(t'-x/v)}E(t')e^{\frac{t'}{c}}dt'\right\}dx\right] \qquad (C–5)$$

Equation (C–5) expresses the discharge as a function of t, and holds on the range $t_0+L/v\leqq t\leqq\infty$ with the additional restriction that $z>\kappa/c$. When $E(t')\equiv 0$ equation (C–5) becomes equation (B–5). For this reason we call it the second generalized form of equation (5). When both $W(x)$ and $E(t')$ are constants, equation (C–5) reduces to equation (32). When $t=t_0+L/v$, equation (C–5) takes the form

$$y=e^{-L/cv}\left[z_0\int_0^L W(x)e^{x/cv}dx\right.$$

$$\left.-\int_0^L \frac{1}{c}W(x)e^{x/cv}\left\{\int_0^{(L-x)/v}E(t')e^{t'/c}dt'\right\}dx\right] \qquad (C–5a)$$

As stated in section 2, about equations (3) and (4), equations (B–3) and (B–4) require no correction for evaporation. Equations (B–6) and (B–9) do, however. The flood crest occurs on the range for which these equations apply. Equation (C–6) below, now to be obtained, holds on the range $t_0\leqq t\leqq t_0+L/v$ with the two additional restrictions that $t>L/v$ and $z>\kappa/c$. By reasoning similar to that used in all previous cases when this range was considered, it follows that for this range the discharge is given by

$$y=\int_{x_0}^L r\left(1-e^{-\frac{1}{c}(t-x/v)}\right)W(x)dx+$$

$$\int_0^{x_0}e^{-\frac{1}{c}(t'-x/v)}\left[z_0-1/c\int_0^{(t'-x/v)}E(t')e^{t'/c}dt'\right]W(x)dx.$$

Since $t'=t-t_0$, $x_0=(t-t_0)v$, and $z_0=r(1-e^{-t_0/c})$ the above expression can be written in the form

$$y=r\left[\int_{x_0}^L W(x)dx-e^{-t/c}\int_0^L W(x)e^{x/cv}dx+\right.$$

$$e^{-(t-t_0)/c}\int_0^{x_0}W(x)e^{x/cv}dx\bigg]-$$

$$\frac{1}{c}e^{-(t-t_0)/c}\int_0^{x_0}W(x)e^{x/cv}\left[\int_0^{t-t_0-x/v}E(t')e^{t'/c}dt'\right]dx. \qquad (C–6)$$

When $E(t')\equiv 0$ equation (C–6) reduces to equation (B–6) and therefore is called the second generalized form of equation (6). When both $W(x)$ and $E(t')$ are constants equation (C–6) becomes equation (31). When $t=t_0$ equation (C–6) becomes equation (B–4a), and when $t=t_0+L/v$ equation (C–6) becomes equation (C–5a). When $t=L/v$ equation (C–6) takes the form

$$y=r\left[\int_{L-t_0v}^L W(x)dx-e^{-L/cv}\int_0^L W(x)e^{x/cv}dx+\right.$$

$$e^{-\frac{1}{c}(L/v-t_0)}\int_0^{L-t_0v}Wxe^{x/cv}dx\bigg]-$$

$$\frac{1}{c}e^{-\frac{1}{c}(L/v-t_0)}\int_0^{L-t_0v}W(x)e^{x/cv}\left[\int_0^{L/v-t_0-x/v}E(t')e^{t'/c}dt'\right]dx. \qquad (C–6a)$$

For the case when the rate of evaporation is independent of the water remaining with the soil, we still must consider the range $t_0\leqq t\leqq t_0+L/v$ with the two additional restrictions that $t_0<L/v$ and $z>\kappa/c$. To save space, however, the equation which applies to this range, which would be numbered (C–9), is not given; this equation (C–9) can be obtained by substituting tv for L in equation (C–6). Similar statements can be made for equation

(C-9) as were made for (C-6) when $E(t') \equiv 0$ and when $E(t')$ and $W(x)$ are constants.

We proceed to find an expression for the time of the maximum discharge, and for the maximum discharge itself. By differentiating equation (C-6) with respect to t, equating to zero, simplifying, and recalling that at the time of the crest $t = t_c$, we finally get

$$re^{t_0 c}\int_0^{t_c v - t_0 v} W(x)e^{x/cv}dx + e^{t c/c}\int_0^{t_c v - t_0 v} E(t_c - t_0 - x/v)W(x)dx$$

$$= r\int_0^L W(x)e^{x/cv}dx +$$

$$\frac{e^{t_0 c/c}}{c}\int_0^{t_0 v - t_0 v} W(x)e^{x/cv}\left[\int_0^{t_0 - t_0 - x/v} E(t')e^{t'/c}dt'\right]dx. \quad (C-7)$$

The solution of equation (C-7) for t_c gives the time of the crest. If the drainage area is of such a shape that the crest may occur on the range $t_0 \leq t \leq t_0 + L/v$ where $t < L/v$ then equation (C-7) should be modified by substituting $t_c v$ for L.

An expression for the maximum discharge is obtained by resorting to the usual device of multiplying equation (C-6) by $e^{t/c}$, then differentiating with respect to t, setting $dy/dt = 0$, and finally multiplying by $ce^{-t/c}$. We thus get

$$y_c = r\int_{t_c v - t_0 v}^L W(x)dx - \int_0^{t_c v - t_0 v} W(x)E(t_c - t_0 - x/v)dx. \quad (C-8)$$

If the drainage area is such, and the rain so short, that the crest may occur before the time $t = L/v$, then $t_c v$ should be substituted for L in the above equation.

The discharge equations thus far derived in this section, and hence the expressions for the maximum discharge and time of the flood crest obtained from them, are valid only if $z_0 > \kappa/c$, and then only till the time that z diminishes to the value κ/c. Discharge equations and expressions for the maximum discharge and time of flood crest will now be obtained for the case when $z_0 < \kappa/c$, that is to say, when the depth of water remaining with the soil at the time the rain stops is not great enough for the rate of evaporation to be independent of it. When this is the case equation (C-2f) gives the rate of run-off for each infinitesimal strip, $W(x)dx$ above the gaging station. By integrating the product, $W(x)zdx$, between the limits 0 and L, taking account of the time required for the water to flow from where it fell as rain to the gaging station, we get

$$y = z_0 \int_0^L W(x)e^{-\frac{1}{c}(t' - x/v) - \frac{1}{\kappa}\int_0^{t' - x/v}\varepsilon(t')dt'}dx. \quad (C-5f)$$

Equation (C-5f) holds on the range $t_0 + L/v \leq t \leq \infty$ with the restriction that $z_0 \leq \kappa/c$. When $\varepsilon(t') \equiv 0$ equation (C-5f) reduces to equation (B-5) and for this reason we call it the third generalized form of equation (5). When $W(x)$ and $\varepsilon(t')$ are constants equation (C-5f) takes the form of equation (26). When $t \to \infty$ equation (C-5f) shows that $y \to 0$, which means that at a sufficiently long time after the rain stops (with no additional rains of course) the stream flow ceases. When $t = t_0 + L/v$ equation (C-5f) takes the form

$$y = z_0 \int_0^L W(x)e^{(L-x)/cv - 1/\kappa\int_0^{(L-x)/v}\varepsilon(t')dt'}dx. \quad (C-5g)$$

Consider now the range $t_0 \leq t \leq t_0 + L/v$ with the further conditions that $t > L/v$ and $z_0 \leq \kappa/c$. Under these conditions and for this range the discharge is given by:

$$y = \int_{x_0}^L rW(x)\left(1 - e^{-\frac{1}{c}(t - x/v)}\right)dx$$

$$+ z_0 \int_0^{x_0} W(x)e^{-\frac{1}{c}(t' - x/v) - \frac{1}{\kappa}\int_0^{t' - x/v}\varepsilon(t')dt'}dx.$$

Since $t' = i - t_0$ and $x_0 = v(t - t_0)$ and $z_0 = r(1 - e^{-t_0/c})$ the above expression simplifies to

$$y = r\left[\int_{x_0}^L W(x)dx - e^{-t/c}\int_{x_0}^L W(x)e^{x/cv}dx\right.$$

$$\left. - (e^{-t_0/c} - 1)\int_0^{x_0} W(x)e^{-\frac{1}{c}(t - t_0 - x/v) - \frac{1}{\kappa}\int_0^{t - t_0 - x/v}\varepsilon(t')dt'}dx\right] \quad (C-6f)$$

When $\varepsilon(t') \equiv 0$ equation C-6f) reduces to equation (B-6), and for this reason we call equation (C-6f) the third generalized form of equation (6). When both $W(x)$ and $\varepsilon(t')$ are constants equation (C-6f) becomes equation (27). When $t = t_0$, equation (C-6f) takes the form of equation (B-4a), and when $t = t_0 + L/v$ equation (C-6f) becomes equation (C-5g). When $t = L/v$ equation (C-6f) has the form

$$y = r\left[\int_{L - t_0 v}^L W(x)dx - e^{-L/cv}\int_{L - t_0 v}^L W(x)e^{x/cv}dx\right.$$

$$\left. + (1 - e^{-t_0/c})\int_0^{L - t_0 v} W(x)e^{-\frac{1}{c}(L/v - t_0 - x/v) - \frac{1}{\kappa}\int_0^{L/v - t_0 - x/v}\varepsilon(t')dt'}dx\right] \quad (C-6g)$$

The last discharge equation to be obtained is one for the range $t_0 \leq t \leq t_0 + L/v$ where $t < L/v$ and $z_0 \leq \kappa/c$. For this range and these additional conditions equation (C-9f) applies; to save space it is not given; it can be immediately obtained from equation (C-6f) by substituting tv for L. Corresponding statements can be made about the form to which equation (C-9f) simplifies when special assumptions are made about $W(x)$ and $\varepsilon(t')$, as were made in regard to equation (C-6f).

We next proceed to obtain expressions for the time of the flood crest and for the maximum discharge. Differentiate equation (C-6f) with respect to t, set this derivative equal to zero, and simplify, recalling that when $dy/dt = 0$ the time of the flood crest is indicated by t_c and $t_c - t_0 = t^*$. On doing this we have:

$$\frac{1}{e^{t_0/c} - 1}\int_{t^* v}^L W(x)e^{x/cv}dx = \int_0^{t^* v} W(x)e^{x/cv - \frac{1}{\kappa}\int_0^{t^* - x/v}\varepsilon(t')dt'}dx$$

$$+ \frac{c}{\kappa}\int_0^{t^* v} W(x)e^{x/cv - \frac{1}{\kappa}\int_0^{t^* - x/v}\varepsilon(t')dt'}\varepsilon(t^* - x/v)dx. \quad (C-7f)$$

The solution of equation (C-7f) for t^* gives the time of the flood crest. If the drainage area is shaped so that the crest may be reached before the time L/v, and if in this case the rain is short enough, then tv should be substituted for L in this equation.

An expression for the maximum discharge is obtained by using the usual device of multiplying equation (C-6f) by $e^{t/c}$, differentiating the equation thus obtained with respect to t, setting $dy/dt = 0$ and finally multiplying by $ce^{-t/c}$. On doing this we get

$$y_c = r\int_{t^* v}^L W(x)dx -$$

$$rce^{-t^*/c}\frac{(1 - e^{-t_0/c})}{\kappa}\int^{t^* v} e^{x/cv - \frac{1}{\kappa}\int_0^{t^* - x/v}\varepsilon(t')dt'}W(x)\varepsilon\left(t^* - \frac{x}{v}\right)dx$$

If the solution of equation (C-7f) for t^* shows that the time of the flood crest occurs before the time L/v, then tv should be substituted for L in equation (C-8f) to obtain the maximum discharge.

Before proceeding further, it will be useful to summarize briefly what has been accomplished thus far in the first three papers of this series. Section I of the first paper and section I of the present paper pertain to *run-off* equations. All the remaining sections, excepting section 5 of the second paper, pertain to *discharge* equations and conclusions which can be drawn from them. Of the sections dealing with discharge equations, the first three in the second paper, and the second in the present paper, consider quite special conditions; while they are instructive and interesting, nevertheless they lack generality and need not be considered further in this brief summary. The second section of the first paper, the fourth section of the second paper, and the present section are developed along closely similar lines. Thus the second section of the first paper derives the five discharge equations (3), (4), (5), (6), and (9). Likewise the fourth section of the second paper derives five discharge equations, bearing the same numbers with the letter B prefixed. The only difference between these two sets of equations is that in the former the drainage area is considered to be rectangular, while in the latter the drainage area may be of any shape encountered in nature. These equations serve to express the discharge from the time the rain begins till the time the discharge has receded to the value it had at the beginning of the rain. Two cases arise, one when the rain lasts long enough for water which fell as rain in that part of the drainage area most remote from the gage to flow to the gage; the other, when the rain did not last this long. The time required for water to flow from the most remote portion of a drainage area to the gage is called the *concentration time*; it has been expressed by L/v. The duration of the rain is expressed by t_0. Now regardless of whether the duration of the rain *exceeds* the concentration time or not, that is, whether $t_0 \gtrless L/v$, four equations are necessary to express the discharge as a function of time from the instant the rain begins till the stream flow has receded to the value it had at the time the rain began. (It is understood, of course, that in this theoretical treatment no additional rains occur after the end of the single one under consideration.) If the duration of the rain exceeds the concentration time, the four equations required are equations (3), (4), (6), and (5) in that order; if the concentration time exceeds the duration of the rain the four equations required are successively (3), (9), (6), and (5). If $t_0 > L/v$, the ranges of the equations are—

equation (3) or (B–3) $0 \le t \le L/v$.
equation (4) or (B–4) $L/v \le t \le t_0$.
equation (6) or (B–6) $t_0 \le t \le t_0 + L/v$.
equation (5) or (B–5) $t_0 + L/v \le t \le \infty$.

If $t_0 < L/v$, the ranges of the equations are

equation (3) or (B–3) $0 \le t \le t_0$.
equation (9) or (B–9) $t_0 \le t \le L/v$.
equation (6) or (B–6) $L/v \le t \le t_0 + L/v$.
equation (5) or (B–5) $t_0 + L/v \le t \le \infty$.

Throughout the second section of the first paper, the fourth section of the second paper, and the present section, much care has been taken to show that at the limits of the ranges of these equations the discharge curve, as a whole, is continuous; in the second article it was pointed out that the rate of discharge curve also is continuous at these points, but that the discharge tendency curve is discontinuous. From the physical nature of the problem, the discharge curve would naturally be expected to be everywhere continuous.

In the present section, discharge equations have been derived in which corrections are made for the evaporation which takes place after the rain stops. If the rate of evaporation is always *dependent* on the water remaining with the soil, as it actually is after many rains, then the treatment of evaporation is rather simple and the discharge curve is expressed by the four equations (B–3), (B–4), (C–6f) and (C–5f) in that order if the duration of the rain exceeds the concentration time, and successively (B–3), (C–9f), (C–6f), and (C–5f) if the concentration time exceeds the duration of the rain. On the other hand if the rate of evaporation is *always independent* of the amount of water that remains with the soil, the discharge curve is expressed by the four equations (B–3), (B–4), (C–6), and (C–5) in that order if $t_0 > L/v$, and successively by (B–3), (C–9), (C–6), and (C–5) when $t_0 < L/v$; but it is doubtful if this case ever occurs in nature. It would seem that the closest approach to such a condition would be when rain falls on ground that is deeply frozen and remains frozen until all surface water from the rain has run off. Actually when the rain lasts so long that z_0 exceeds κ/c and makes the rate of evaporation independent of the amount which remains, the rate of evaporation remains independent of this amount only so long as z exceeds κ/c. For this reason more than four equations are required to express the discharge curve. If $\bar{t}$ represents the time measured from the instant the rain stops, at which the rate of run-off recedes to the value κ/c, then for any small parcel of ground the rate of run-off is given by equation (C–2) during the interval $0 \le t' \le \bar{t}$, and by equation (C–2f) after having replaced z_0 by κ/c during the interval $\bar{t} \le t' \le \infty$. Whenever the rain lasts long enough to make z_0 exceed κ/c, six equations instead of four are necessary to represent the discharge curve.

The additional equations necessary for a complete treatment will not be derived; only an explanation of how they may be obtained, and the ranges of their applicability, are given. All the discharge equations in the three sections here summarized can be placed in one of three classes. One class comprises all those equations derived by integrating equation (1) over the drainage area, with, of course, consideration of the time required for the water to flow from where it fell as rain to the gage. In the second class are placed all those equations derived by integrating any one of the run-off equations (2), (C–2), or (C–2f) over the drainage area. To obtain any discharge equation which belongs to either of these classes, only one integration with respect to x is required. In the third class are placed those equations for which two integrations with respect to x were necessary. Equation (1) must be integrated over the proper portion of the drainage area, and one of the three run-off equations (2), (C–2), or (C–2f) must be integrated over the remainder of the drainage area. The flood crest always occurs on the range in which a discharge equation of this third class is applicable.

When z is expressed for a certain interval by equation (C–2), and for a later interval by equation (C–2f) after z_0 is replaced by κ/c, two cases arise, one when $\bar{t} > L/v$ and the other when $\bar{t} < L/v$. These two cases are somewhat analogous to those which arise according as the concentration time exceeds or is less than the duration of the rain. When $\bar{t} > L/v$ it will never be necessary to perform more than two integrations with respect to x to obtain an expression for the discharge. One integration must be made on equation (C–2) over the proper portion of the drainage area and the other on equation (C–2f) over the remainder, in each case taking account of the time required for water travel. When $\bar{t} < L/v$ it is necessary

over part of the range to perform three integrations with respect to x, one on equation (1), another on equation (C-2) and a third on equation (C-2f), each over the proper portion of the drainage area with due regard for the time required for water travel.

When $\bar{t} > L/v$ equations (9) and (6) require no change; equation (C-5) holds only on the range $t_0 + L/v \leqq t \leqq t_0 + \bar{t}$; a new equation, not here derived, applies on the range $t_0 + \bar{t} \leqq t \leqq t_0 + \bar{t} + L/v$; and equation (C-5f), after z_0 has been replaced by κ/c, applies on the range $t_0 + \bar{t} + L/v \leqq t \leqq \infty$.

When $\bar{t} > L/v$ equation (B-6) holds only on the range $t_0 \leqq t \leqq \bar{t} + t_0$; equation (B-9) holds only on the range $t_0 \leqq t \leqq \bar{t}$; a new equation requiring three integrations with respect to x, applies on the range $t_0 + \bar{t} \leqq t \leqq t_0 + L/v$; another new equation applies on the range $t_0 + L/v \leqq t \leqq t_0 + L/v + \bar{t}$; and finally equation (C-5f), after z_0 has been replaced by κ/c, applies on the range $t_0 + \bar{t} + L/v \leqq t \leqq \infty$.

Consider now the expressions for the maximum discharge. It can be shown that for each value of t the values of y given by equations (C-6) and (C-6f) respectively, are less than the corresponding value of y given by equation (B-6). Hence evaporation reduces the discharge, and it is obvious that this should be so. It readily follows that the maximum discharge as given by equations (C-8) and (C-8f), respectively, is less than the maximum discharge given by equation (B-8).

It was explained in the second article that equation (B-8) has a physical interpretation. Equation (C-8) has a similar interpretation. This physical explanation can be most simply stated when the rate of evaporation, $E(t')$, is constant; in this case the maximum discharge can be described as equal to the discharge in a steady state from that portion of the drainage area situated above the equal water travel line which corresponds to the distance water travels from the time the rain stops till the time of the flood crest, diminished by the volume of rate of evaporation from the remainder of the drainage area at the time of the crest. It is clear, in the light of the explanation previously given for equation (B-8), why the maximum discharge has this value. If the rate of evaporation is not constant, then the maximum discharge is equal to the discharge in a steady state from that portion of the drainage area situated above the equal water travel line which corresponds to the distance water travels from the time the rain stops till the time of the flood crest, diminished by the volume of rate of evaporation obtained by integrating the rate of evaporation over the remainder of the drainage area at the time of the crest diminished by the time required for water to flow to the gage from where it fell as rain.

In equation (C-8f), which gives an expression for the maximum discharge when the rate of evaporation is dependent on the amount of water remaining with the soil, we can replace $r(1 - e^{-t_0/c})$ by z_0, and then replace $z_0 e^{-\frac{1}{c}(t^* - x/v) - \frac{1}{\kappa}\int_0^{t^*} -x/v \, \mathcal{E}(t')dt'}$ by $z(t^* - x/v)$. Then $cz(t^* - x/v)$ can be replaced by $\phi(t^* - x/v)$, which function represents the depth of water remaining with the soil. Finally $\frac{1}{\kappa}\phi(t^* - x/v)\mathcal{E}(t^* - x/v) = E(t^* = x/v)$. Hence equation (C-8f) admits of exactly the same physical explanation as equation (C-8).

The final topic to be considered is the effect of evaporation on the time of the flood crest. This effect is not at all as obvious as the effect of evaporation on the maximum discharge. It is not obvious whether the effect of

evaporation is to advance or to retard the time of the flood crest.

The difference between equation (B-6) and equation (C-6) is

$$1/c \int_0^{t'r_0} W(x) e^{-\frac{1}{c}(t' - x/v)} \left[\int_0^{t' - x/v} E(t') e^{t'/c} dt' \right] dx$$

Since all functions in the integrand are positive, this expression is an increasing function of t. Hence we conclude that the effect of evaporation is to advance the time of the crest; that is to say, the crest comes sooner when evaporation takes place than it does when evaporation does not occur after the rain stops. However, from the form of equation (C-7) it is evident that evaporation does not affect the time of the flood crest by a large amount.

The difference between equation (B-6) and (C-6f) is:

$$r(1 - e^{-t_0/c}) \int_0^{t'r} W(x) e^{-\frac{1}{c}(t' - x/v)} \left(1 - e^{-\frac{1}{\kappa}\int_0^{t' - x/v} E(t')dt'} \right) dx.$$

This is an increasing function of t; hence whether the rate of evaporation is dependent or independent of the amount of water that remains with the soil, the effect of evaporation is to advance the time of the crest.

SECTION 4: THE DIURNAL VARIATION OF THE RATE OF RUN-OFF

It is well known that in moderately dry weather a rather pronounced diurnal variation is often observed in the discharge of very small streams. This diurnal variation obviously cannot be observed in very dry weather, since small streams then become dry, although undoubtedly the cause which brings about the variation is still present; the variation in streamflow may be present also when the streams are flowing at more than average stages (that is, in moderately wet weather), but if so its amplitude is not so great and hence the phenomenon is not easily observed, although again the cause undoubtedly is present. However, no diurnal variation is observed in the larger streams; and it is the object of this section to explain why this variation is absent in the larger streams, and to show that the effect of this factor can be neglected in forecasting the time and magnitude of flood crests.

It is evident that there must be a diurnal variation in the rate of evaporativity; however, it is clear from equation (C-2f), that regardless of how great the amplitude $E(t')$ may be in the rate of evaporativity, there can be no variation whatever in the rate of run-off, z. Moreover, from equation (C-2) it is evident that no variation in the rate of evaporation, $E(t')$, however great, can cause any variation in the rate of run-off, z. Since a diurnal variation is actually observed in the rate of run-off it is obvious then that equations (C-2) and (C-2f) do not represent all the conditions which occur in Nature.

It seems plausible that in the case of soils not covered with vegetation, the only forces acting on the water in the soil are gravity, the molecular attraction of the soil particles for the water molecules, and the vapor tension due to air circulating between the soil particles. Under this condition it seems that the equations given in section 1 represent the rate of run-off with a reasonable degree of accuracy. When plants are present, however, there is an additional force to be considered. The transpiration of plants seems to be equivalent to a force on the soil

water opposite to that of gravity. This influence causes the diurnal surge or variation in the rate of run-off.

Take the usual expression for the amount of water remaining with the soil at time t',

$$A\phi(t') = AR - F(t_0) - A\int_0^{t'} z\,dt' - \int_0^{t'} AE(t')dt',$$

and in accordance with the second fundamental assumption replace $E(t')$ by $\frac{1}{\kappa}\phi(t')\mathcal{E}(t')$. If $\theta(t')$ represents the magnitude of the force acting opposite to gravity which transpiration causes at time t', then the third fundamental assumption can be expressed as

$$cz = \frac{k\phi(t')}{\theta(t')},$$

where k is a constant of proportionality, whence we have

$$\frac{Ac}{k}z\theta(t') = AR - F(t_0) - A\int^{t'} z\,dt' - \int_0^{t'}\frac{A}{k\kappa}cz\theta(t')\mathcal{E}(t')dt';$$

then by differentiating we have

$$\frac{Ac}{k}\theta(t')\frac{dz}{dt'} + \frac{Ac}{k}z\frac{d}{dt'}\theta(t') = -Az - a\frac{Ac}{k\kappa}\theta(t')\mathcal{E}(t'),$$

or on simplifying,

$$\frac{dz}{z} + \frac{d\theta(t')}{\theta(t')}dt' = -\frac{kdt'}{c\theta(t')} - \frac{1}{\kappa}\mathcal{E}(t')dt'$$

On integrating this last expression between the limits 0 and t' and then solving for z we get

$$z = z_0\frac{\theta(0)}{\theta(t')}e^{-\int_0^{t'}\left[\frac{k}{c\theta(t')} + \frac{1}{\kappa}\mathcal{E}(t')\right]dt'} \qquad (36)$$

It should be emphasized that z_0 in equation (36) is not given by $r(1 - e^{-t_0/c})$ as it has been in all previous equations, but instead $z_0 = \frac{k}{\theta(0)}r(1 - e^{-t_0/c})$.

The function $\theta(t')$ is periodic in ordinary weather with a period of 1 day. As this function increases, the rate of run-off decreases, and vice versa. The rate of evaporativity $\mathcal{E}(t')$ may be, and usually is, periodic also, but its oscillations do not produce any variation in the rate of run-off.

To obtain an expression for the discharge, equation (36) is integrated over the drainage area with due regard to the time required for water to flow from where it fell as rain to the gage. This is done by replacing t' by $t' - x/v$ and integrating with respect to x between the proper limits. It is evident that the exponential in equation (36) does not change rapidly with a change in t'. Therefore if $\theta(t' - x_1/v)$ corresponds to (or, rather, equals) $\theta(t' - x_2/v)$, the times $t' - x_1/v$ and $t' - x_2/v$ differing by 1 day, then on integrating equation (36) between the limits x_1 and x_2, the periodicity disappears. As the velocity of the water is not very great when streams are low, the distance which water will travel in 1 day is not relatively great, and therefore the stream at the outlet of a drainage area of moderate length does not display a noticeable diurnal variation in its discharge. Moreover the other irregularities of the drainage area tend to mask this phenomenon.

SECTION 5: HYDROGRAPHS OF DISCHARGE

Three hydrographs of hypothetical conditions are here computed and shown in the figures. Each hydrograph represents the discharge with evaporation neglected as well as with evaporation considered. Obviously the lower branch of each curve represents the discharge corrected for evaporation.
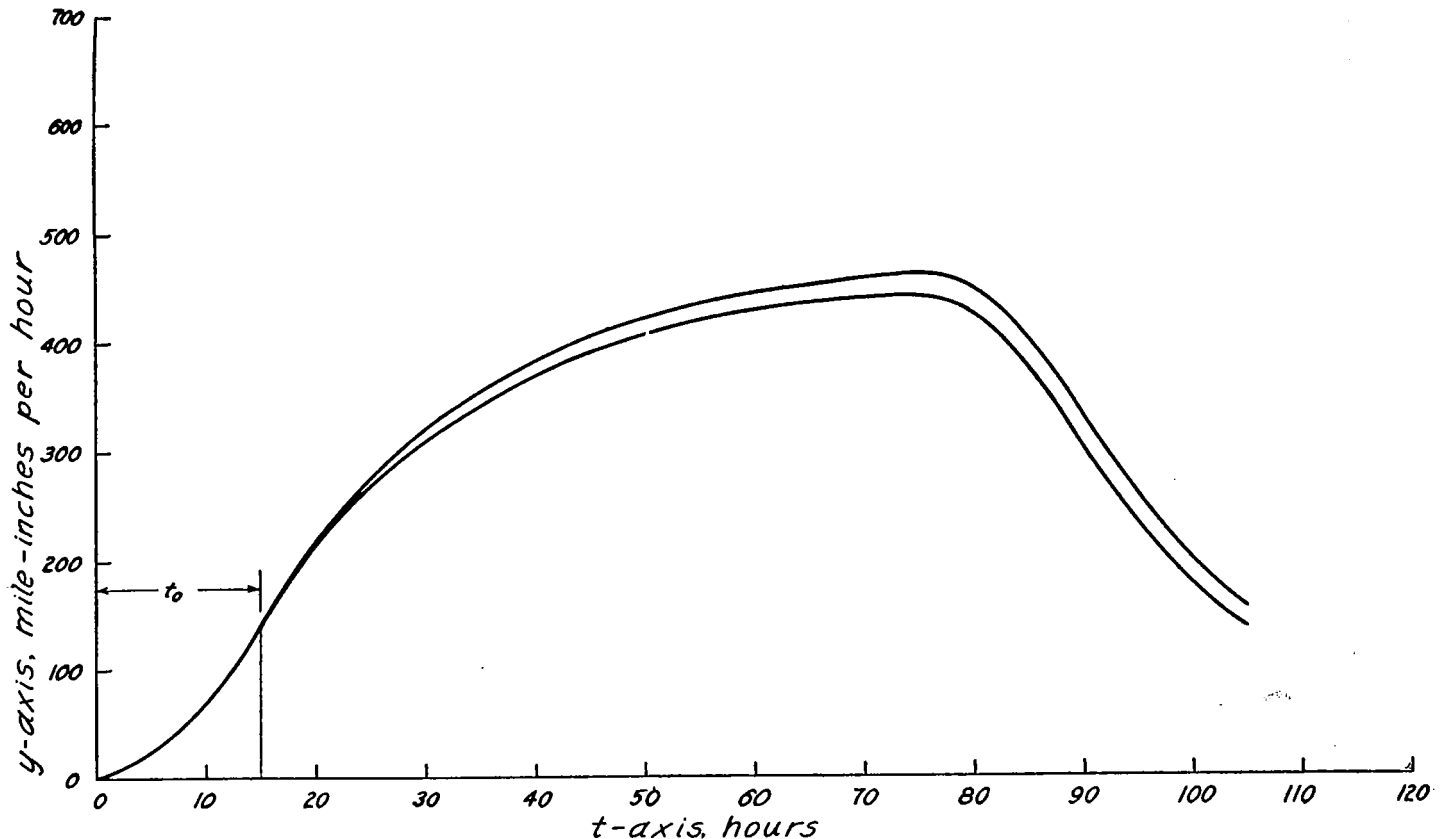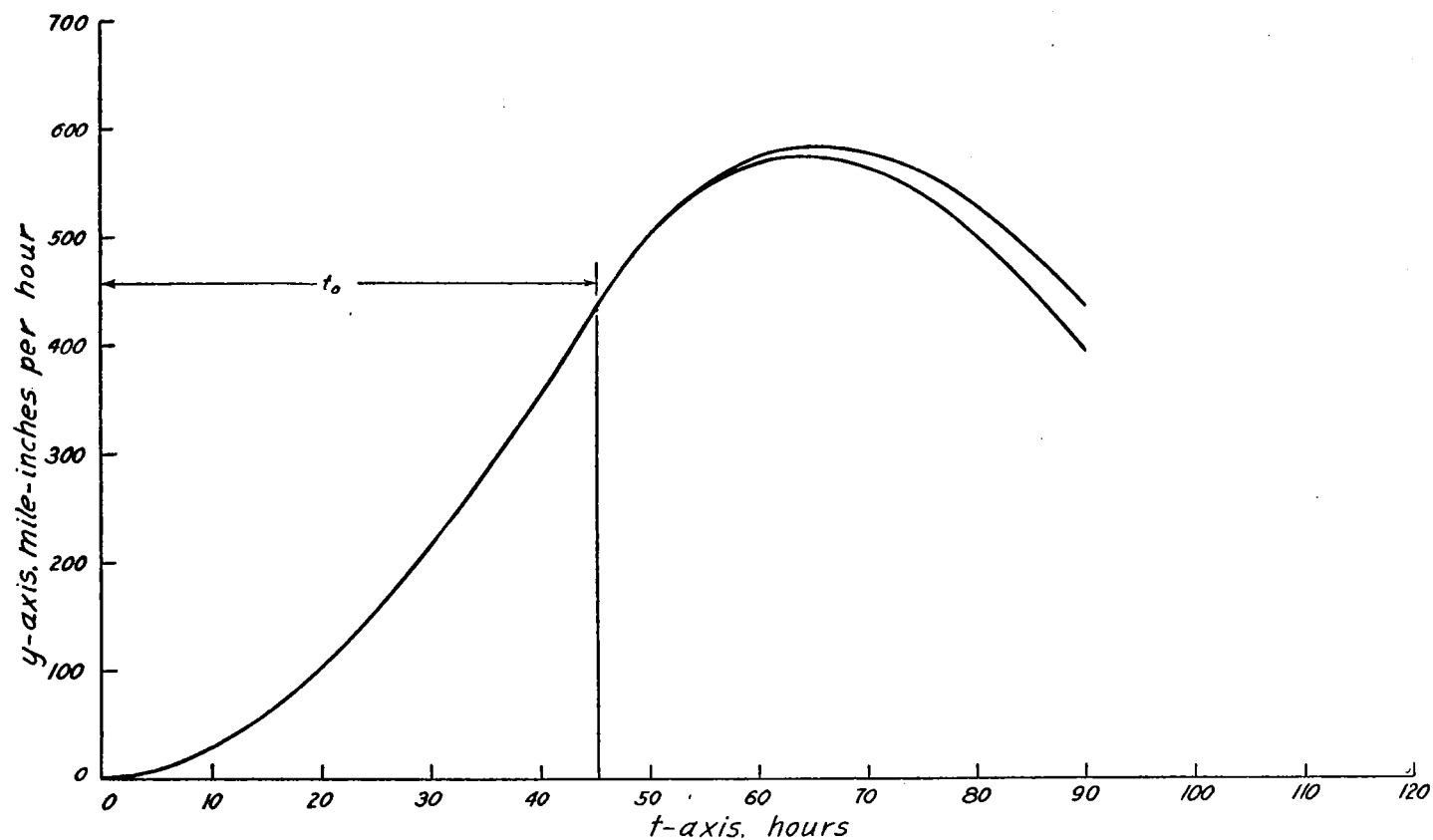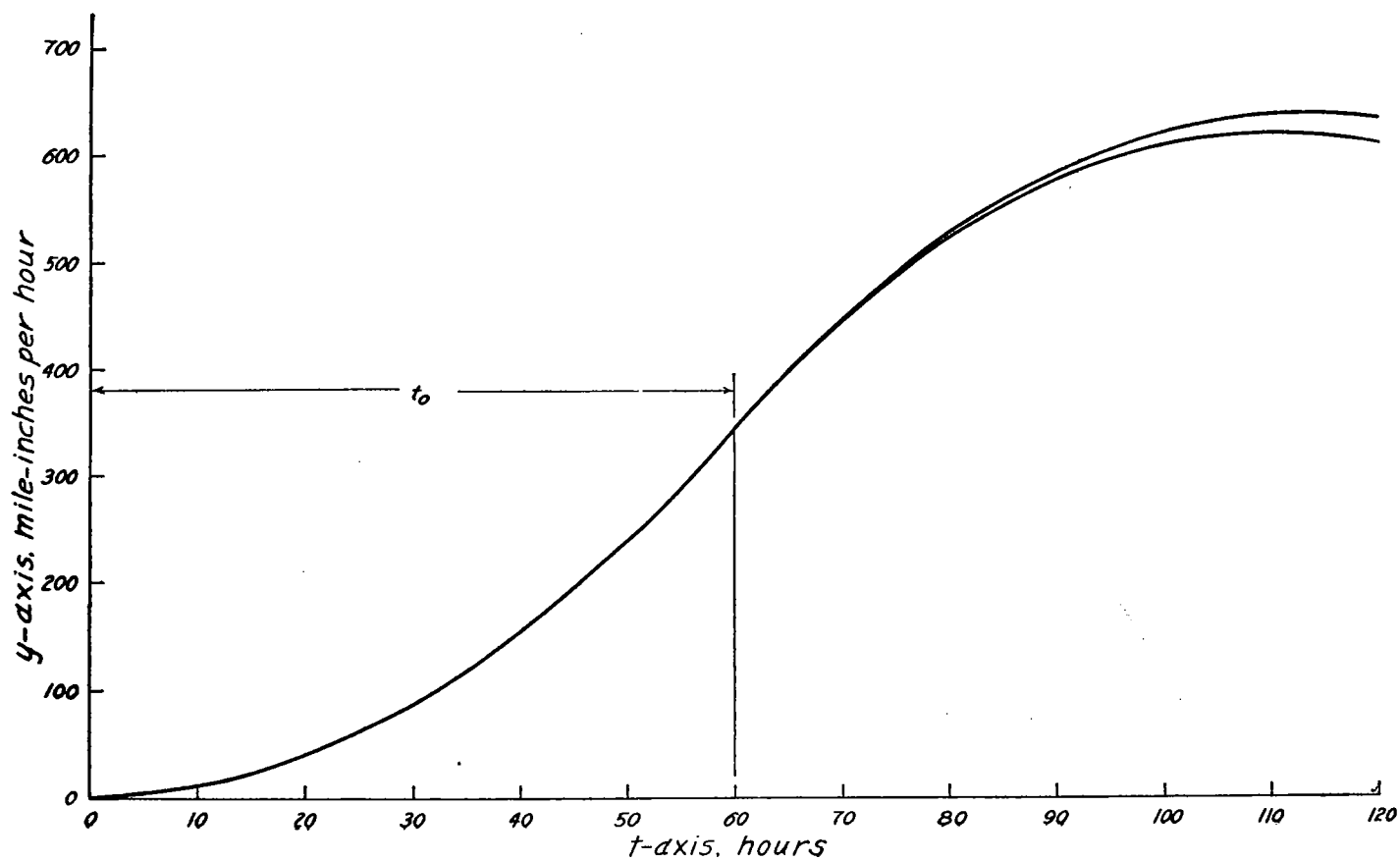

Figure 10.

FIGURE 11.

FIGURE 12.

Figure 10[3] shows the effect of evaporation on the discharge for a relatively short rain. For this figure the values of the constants are as follows: $t_0 = 15$ hours, $L = 300$ miles, $W = 40$ miles, $r = 0.20$ inch per hour; $E = 0.015$ inch per hour; $v = 4$ miles per hour; $c = 20$ hours; $t = 20$ hours and $\varepsilon = 0.015$ inch per hour. The values of E and ε are very high. Such high values are purposely assumed so as to form a conclusion of the maximum effect evaporation may have in diminishing the height of the crest. For this same reason W is purposely made large so that the time after the rain stops may be long during which evaporation may take place. The maximum discharge with evaporation neglected is 463.4 mile-inches per hour and 440.1 when corrected for evaporation. This is an extreme case and the effect of evaporation is only about 5 percent. In this case the effect of evaporation on the time of the crest is extremely minute.

Figure 11 was constructed from different values of the

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[3] The figures are numbered consecutively with those in papers I and II.

constants. In this case the values were purposely chosen to show the maximum effect evaporation may have on the time of the crest. For this figure $t_0 = 45$ hours, $L = 180$ miles, $W = 40$ miles, $r = 0.15$ inch per hour, $E = 0.015$ inch per hour, $v = 4$ miles per hour; $c = 40$ hours. It is further assumed that $t > t_0 + L/v$. When evaporation is neglected, the time of the crest is 65 hours 38 minutes from the beginning of the rain, and is 64 hours 9 minutes when evaporation is considered.

Figure 12 was constructed to show the effect of evaporation on discharge when there is no surface run-off. Here $c'' = 0$ and $c' = c = 500$ hours, $t_0 = 60$ hours, $L = 240$ miles, $W = 200$ miles, $r = 0.05$ inches per hour, $\varepsilon = 0.005$ inches per hour, $v = 4$ miles per hour. The effect of evaporation in this case is small also. Here the width of the drainage area has purposely been chosen 5 times as large as in the two earlier cases because of the scale in the figures.

PRELIMINARY REPORT ON A STUDY OF ATMOSPHERIC CHLORIDES

By WOODROW C. JACOBS

[University of California, Scripps Institution of Oceanography, La Jolla, Calif., August 1936]

An investigation of the salt[1] content of the air was begun at the Scripps Institution in January 1936. The primary purpose of the investigation, at the time it was initiated, was to attempt to prove that the formation of shallow haze or fog (salt haze), peculiar to coastal regions, is due to the presence of comparatively large salt particles or droplets of concentrated sea water suspended in the air. However, a later survey of the available literature in the field of colloid meteorology revealed that such an investigation might serve to fill a gap in our present knowledge of the sources of condensation nuclei. It was felt that a knowledge of the probable sizes of the particles and their effectiveness as condensation nuclei together with a determination of the salt content of the air would make deductions possible regarding the importance of the sea as a source of atmospheric nuclei.

CONDENSATION NUCLEI AND ATMOSPHERIC NUCLEATION

Since the experiments of John Aitken and C. T. R. Wilson, it has been known that condensation of water vapor will not occur at ordinary humidities in air which does not contain colloid particles to act as condensation nuclei. Lord Kelvin (1) explained this as an effect of the increased vapor pressure over a convex surface, which would render condensation impossible in the absence of a nucleus. Later experiments, however, indicate that it is probably a question largely of true metastability. Studies of the nature and effects of these nuclei have been made by numerous investigators, the results of whose researches may be found in many published papers; yet, very little is known concerning the origin of those suspensoids which are active in the atmospheric condensation processes.

Dust particles were at first considered to be the active nuclei; but subsequent investigations by Wigand (2), Boylan (3), Owens (4) and others have proved beyond a doubt that these neutral particles will act only at enormous degrees of supersaturation. Aggregates of the air molecules and complex water molecules may also be eliminated from further consideration on the same grounds. Molecular ions have been considered, but C. T. R. Wilson proved that they were effective only under the extreme

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[1] The term "salt", herein used, is not limited to $NaCl$.

conditions imposed within his cloud chamber. He found that to produce condensation on negative ions a relative humidity of 420 percent is required, on positive ions 790 percent. Appreciable supersaturation is necessary before condensation will take place on even those large, slow-moving, charged particles, the Langevin ions; therefore, it seems as though we may safely disregard these, too, as being effective in an atmosphere where such a state seldom exists. In fact, as stated by Willett (5):

> The whole trend at present, in the light of increasing observational data on the conditions actually prevailing in clouds where condensation is taking place, is to postulate an ever smaller degree of supersaturation in these processes.

It has been observed many times that fogs and clouds frequently form at relative humidities well below 100 percent, which condition seems to be the rule in some localities rather than the exception. On the other hand, even very slight degrees of supersaturation are seldom found in fogs and clouds and then, usually, only under extreme conditions of cooling. That this condition is more frequent at high levels, say in cirrus clouds, or in cumulo nimbus clouds, may be true. Evidence at the present time points to the importance of the hygroscopic aerosol in the condensation process. It is well known that the vapor pressure is lower over a solution or a hygroscopic substance than over a plane water surface, hence, such particles in the atmosphere, coupled with a large curvature, constitute extremely effective nuclei for condensation.

As to the origin of these particles, there is considerable difference of opinion. According to Bennett (6), there are two obvious possibilities—the sea and chimneys. It would be expected that the sea would contribute most of those nuclei effective over and near the sea, while those resulting from combustion are no doubt of extreme importance near such sources of pollution. However, there is no evidence indicating that there has been any great change, except locally, in the balance of nature in this respect since the rise of industrialism. Therefore, Köhler (7), Simpson (8), Melander (9), Ludeling (10) and others believe that the sea is the primary source; a reasonable conclusion when it is considered that five-sevenths of the earth's surface is water. In support of such a theory, Köhler from his analyses of rains, snows, and rime, found an almost constant chloride content even at great distances